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Shallow Binding in Lisp 1.5

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Shallow binding is a scheme which allows the value of a variable to be accessed in a bounded amount of computation. An elegant model for shallow binding in Lisp 1.5 is presented in which context-switching is an environment tree transformation called *rerooting*. Rerooting is completely general and reversible, and is optional in the sense that a Lisp 1.5 interpreter will operate correctly whether or not rerooting is invoked on every context change. Since rerooting leaves *assoc[v,a]* invariant, for all variables *v* and all environments *a*, the programmer can have access to a rerooting primitive, *shallow[]*, which gives him dynamic control over whether accesses are shallow or deep, and which affects only the speed of execution of a program, not its semantics. In addition, multiple processes can be active in the same environment structure, so long as rerooting is an indivisible operation. Finally, the concept of rerooting is shown to combine the concept of shallow binding in Lisp with Dijkstra's *display* for Algol and hence is a general model for shallow binding.

Key Words and Phrases: Lisp 1.5, environment trees, FUNARG's, shallow binding, deep binding, multiprogramming, Algol display

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1. Introduction

A severe problem in Lisp 1.5 [6] systems is the amount of time required to access the value of a variable. This problem is compounded by Lisp's choice of "fluid" or "dynamic" scoping for nonlocal (free) variables, wherein a procedure's free variables are considered bound in the environment of the caller (or the caller's caller, etc.). On the other hand, Algol and most other computer languages (with the major exception of APL) use a *lexical* scoping rule in which the free variables of a procedure are bound in the block which lexically (textually) embraces the procedure. Dynamic scoping leads to environment trees at run-time which are "tall and thin" rather than "short and bushy" because the topology of the environment tree follows that of the run-time stack rather than the block structure of the source text. Since the length of time to access the binding of a variable in the original Lisp 1.5 implementation [6] is proportional to the distance in the tree from the current environment to the place where that variable is bound, this time can be quite large for the tall environment trees actually encountered. For example, accessing a global variable from the bottom of a deep recursion requires time proportional to the depth of the recursion.

The Maclisp interpreter [7] solves this problem through a scheme called "shallow binding". In this scheme, each variable has a *value cell*, which can be accessed in constant time, and the bindings associated with the current environment are kept in these value cells so that they can be accessed without any search. Whenever a context change occurs, such as when calling or returning from a function, some of the value cells must be changed or restored. Since Lisp uses dynamic variable scoping, the changes in the environment mirror those in the return-point stack, so Maclisp saves the old bindings from the value cells on a parallel stack called the *specpdl*[1] when calling a procedure and restores them from the specpdl when returning from a procedure. As a result, variable access

[1] *Spec*ial variable *p*ush *d*own *l*ist.

time in Maclisp is constant and context-switching time is unbounded, whereas in so-called deep binding systems like Lisp 1.5, the access time is unbounded and the context-switching time is constant. Thus, a tradeoff exists between the time needed to access the binding of a variable and the time needed to switch contexts, and whenever variable access is more common than context switching, shallow binding is more efficient.

However, the scheme used in Maclisp cannot be used to implement shallow binding in Lisp 1.5, because it does not handle function-producing functions (so-called *upward* FUNARG's [8]) correctly. In fact, no stack-based environment structure can do so because using LIFO allocation discipline for Lisp environment nodes can lead to the premature unbinding of the free variables required by these upward FUNARG's. On the other hand, our model for shallow binding keeps a *tree* environment structure and allocates environment nodes in the heap, and it is therefore capable of handing full FUNARG's.

2. The Environment Tree Rerooting Model

In our model, the environment structure consists of an *environment tree*, a current environment ("you are here") *pointer*, and a global *value cell* for each variable name. An environment tree node consists of a pair of pointers *<binding,parent>*, where *binding* is a pair *<name,value>*, and *parent* is another environment node or the distinguished pointer NIL. The interpretation of an environment node *E'* = *<<name,value>,E>* is that the value of a variable in *E'* is the same as the value of that variable in *E*, unless it is the variable *name*, in which case it has the value *value* in the environment *E'*. The value cell of each variable, on the other hand, contains the *default* binding of that variable, i.e. the binding to be used if the variable is not bound in any environment node along the "parent" path from the current environment node to the root in the environment tree.

The system is initialized so that the current environment pointer points to the top level environment, an environment node having a dummy binding and a NIL parent, thus identifying it as the current root. The value cell of each variable is initialized to its top level binding, i.e. the pair *<variable-name,top-level-value>*, where the redundant *variable-name* simplifies programs presented later. If the variable has no top level value, *top-level-value* is the distinguished indicator "UNBOUND", which causes a trap if the interpreter tries to return it as the value of a subexpression.

The algorithm for accessing a variable is: 1) search the environment tree from the current environment to the root, and if the variable name is found, use the associated binding; else 2) use the binding in the value cell. Figure 1 gives Mlisp-like code for a version of Lisp's assoc function which implements this algorithm.

Fig. 1. Variable lookup mechanism.

```
assoc[v,a] =                                  // Return the binding of v in the environment a.
  [parent[a]=NIL       -> vcell[v];
   v=name[binding[a]] -> binding[a];
   T                  -> assoc[v,parent[a]]]

parent = CDR                                  // Parent, binding, and name are new names
binding = CAR                                 // for cdr, car, and car, respectively.
name = CAR
```

`vcell[y]` returns the value cell of variable *y*, which is a cons cell having the structure `(y . value[y])`, in constant time.

So far we have described a conventional deep binding interpreter in which the *global* or *top level* value of each variable is kept in its value cell. In order to convert this system into a *shallow* binding system, we need only make sure that the distance from the current environment to the root in the environment tree is *short*; i.e. *we make sure that the current environment is always the root*. In this case, the search is always trivial and is eliminated.

The way that we do this is to *reroot* the environment tree at each context-switch,[2] so that the context to be switched to becomes the new root. The rerooting algorithm is quite simple. Suppose that *E* is the current root of the environment tree and *E′* is any other node in the tree which we wish to become the new root. Since *E* is the current root of the tree, the "parent" path in the tree from *E′* will terminate at *E*. We make two passes over that path, one forward pass and one backward pass. On the forward pass, we go from *E′* to *E*, stacking the nodes as we go. On the backward (relative to the original path) pass, we go from *E* back to *E′*, reversing the binding links and exchanging the values in the association pairs with those in the value cells. This has the effect of terminating with the proper values for *E′* in the value cells.

Figure 2 presents a modified *assoc* function which gets a variable binding directly from its value cell without any search and a modified *apply* function which reroots the environment tree whenever a context switch is made. These functions are intended to work with a Lisp interpreter like that found in the Lisp 1.5 Programmer's Manual [6].

Fig. 2. Continuous shallow-binding interpreter.

```
assoc[v,a] = vcell[v]                // Return the binding of v in the current environment a.

apply[f,x,a] =                               // Apply function f to arguments x in environment a.
  [atom[f]        ->[f=CAR  -> caar[x];
                     f=CDR  -> cdar[x];
                     f=CONS -> cons[car[x],cadr[x]];
                     f=ATOM -> atom[car[x]];
                     f=EQ   -> car[x]=cadr[x];
                     T      -> apply[eval[f,a],x,a]];
   car[f]=LAMBDA -> prog1[eval[caddr[f],reroot[pairlis[cadr[f],x,a]]],reroot[a]];
   car[f]=FUNARG -> prog1[apply[cadr[f],x,caddr[f]],reroot[a]]]

pairlis[vars,vals,a] =             // Extend environment a with new bindings of vars to vals.
  [vars=NIL -> a;
   T        -> cons[cons[car[vars],car[vals]],pairlis[cdr[vars],cdr[vals],a]]]

reroot[a] =                                                          // Make a the new root.
  [parent[a]=NIL -> a;                                             // a is already the root.
   T             -> onestep[a,reroot[parent[a]]]]

onestep[new,old] =                                       // new is new root; old is old root.
  prog[[];
       parent[new] := NIL,                                          // Reverse binding link.
       parent[old] := new,
       binding[new] :=: binding[old],
       binding[old] :=: vcell[name[binding[old]]];       // Exchange binding with value cell.
       return[new]]

progi[<e1>, <e2>, …, <en>] evaluates <e1>, <e2>, …, <en> in order and returns the value of
<ei>.
":=:" indicates an exchange operation.
```

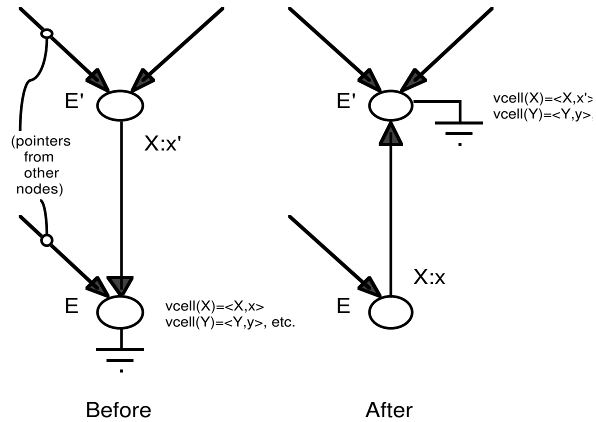
We outline a proof of correctness for our model in two stages. We show that the root of the environment tree can be moved one step while preserving *assoc[v,a]* for all variables *v* and all environments *a*, and then we exhibit a rerooting procedure which finds an appropriate directed path for the root movement and moves it by iterating the single-step method.

To prove that single-step movement is possible, consider the two nodes *E* and *E′* of the environment tree in Figure 3. Before the transformation, *E* is the root and *E′* is some other node exactly one step from the root. Now since *E′* is only one step from *E*, it differs from *E* in the value of exactly one variable, which we can assume to be *"X"* without loss of generality. Now since *E* is

[2] A context switch is required for 1) a function call; 2) a return from a function call; 3) a nonlocal goto; 4) an interrupt (if handled by a Lisp function); or 5) a return from an interrupt.

the root, *X*'s binding to *x* in that context is stored in *X*'s value cell. The directed link from *E′* to *E* encodes the fact that the binding of *X* in *E′* is different, namely *x′*.

Fig. 3. The one step environment transformation.



Now to reroot the tree at *E′*, we need only rearrange values and pointers in such a way that the bindings returned by *assoc* for both *E* and *E′* are not affected, yet *E′* becomes the new root. The appropriate change is to invert the link between *E′* and *E*, and exchange the binding on the link (<*X*,*x′*>) with the binding in *X*'s value cell (<*X*,*x*>). This accomplished, we notice that *assoc* will still return the correct binding for *X* in both *E′* and *E*. Furthermore, the bindings of other variables in *E* and *E′* have not been affected, since those bindings remain in their value cells. Finally, the bindings of variables in contexts which point to *E′* or *E* have not changed, since *E′* and *E* retain their identities in the values they assign to variables. Thus, single step root movement is possible, and can be iterated to make any node in the tree the root.

However, if we desire to make a root from an arbitrary node, we must interpolate a path from the root to that node since the root may move only one step at a time. Since the environment structure is a tree, such a path always exists and is unique. The rerooting procedure described above traces this path forwards from the arbitrary node to the root and iterates the single-step transformation on the way back.

Since the parent links must be reversed on the second pass anyway, we can reverse them on the first pass and avoid the use of a stack. Figure 4 illustrates this technique. This program for rerooting has much in common with the Deutsch-Schorr-Waite stackless list-tracing technique [5].

Fig. 4. Stackless rerooting.

```
reroot[a] =                                  // Make a the new root using Deutsch-Schorr-Waite.
 prog[[r,b,p]
      r := nreverse[a],                                           // Reverse a as in D-S-W.
      b := binding[r],                              // Save dummy binding from the old root.
      p := r,                                                   // Initialize p for loop.
   while parent[p] ≠ NIL
   do prog3[binding[p] := binding[parent[p]],
            binding[p] :=: vcell[name[binding[p]]],
            p := parent[p]],
   binding[a] := b,
   return[a]]

nreverse[a] = nreconc[a,NIL]              // Destructively reverse list a, by flipping CDR pointers.

nreconc[a,b] =
  [a=NIL -> b;
   T     -> nreconc[cdr[a],rplacd[a,b]]]

rplacd[x,y] = prog2[cdr[x]:=y,x]
```

The continuous shallow binding interpreter presented above is equivalent to the Maclisp interpreter for that class of Lisp programs which do not have FUNARG's, in the sense that the tree environment it creates consists of a single *path* (tree with no branches) which is isomorphic to the Maclisp *specpdl*. However, our implementation of the model, being simple for pedagogic reasons, is not necessarily the most efficient way to perform shallow binding; e.g. since our environment nodes live in the heap rather than on a stack, they are more costly to reclaim.

3. Storage Requirements of Shallow Binding *vis á vis* Deep Binding

Our shallow-binding model requires a value cell for every variable used by a program in addition to the storage required by the accessible environment tree nodes. Therefore, the storage requirements are at least superficially the same as for a deep-binding model with global value cells. However, one could conceive that, due to the pointer rearrangement in our model, the two models might differ in the details of when particular environment nodes (and hence the values to which they point) can be reclaimed by the garbage collector. It so happens that this is *not* the case; the models are equivalent in the sense that the time-varying set of accessible nodes is precisely the same in both models. The proof is beyond the scope of this paper, but depends on the fact that rerooting preserves the in-degree of all environment nodes, except for the old and the new roots. This accessibility equivalence to deep binding is a nontrivial property of our shallow-binding model; some other models, such as deep binding with a *cache cell* for each variable [1,2,10], may retain pointers to nodes long after they would have become inaccessible in the deep-binding model.

4. Casual Rerooting

Suppose now that we have a standard deep binding (nonrerooting) Lisp 1.5 interpreter which has been augmented by the *shallow[]* primitive (see Figure 5). When this primitive is invoked, the environment tree will be rerooted to the current environment. Rerooting transforms the environment tree in such a way that *assoc[v,a]* is left invariant; therefore the interpretation proceeds correctly, but the length of variable searches is reduced. In other words, the interpreter may *casually* reroot instead of rerooting at every context switch without affecting the semantics of the language.

Fig. 5. Casual shallow-binding interpreter.

```
assoc[v,a] =                                      // The original deep-binding assoc.
  [parent[a]=NIL      -> vcell[v];                // Is a the root?
   v=name[binding[a]] -> binding[a];
   T                  -> assoc[v,parent[a]]]

apply[f,x,a] =
 [atom[f]       -> [f=CAR -> caar[x];
                    ...
                    f=SHALLOW -> prog2[reroot[a],T]    // Return T as value of shallow[]
                    T -> apply[eval[f,a],x,a]];
  car[f]=LAMBDA -> eval[caddr[f],pairlis[cadr[f],x,a]];
  car[f]=FUNARG -> apply[cadr[f],x,caddr[f]]]
```

This flexibility to run some functions shallow and some deep may reduce overall running time. For example, a tight loop might run faster when shallow bound, while an interrupt handler might run better deep bound, since rerooting can be time consuming.

Lisp 1.5 can also be augmented with primitives for multiprocessing such as *fork*, *P*, and *V*. Several processes can be active in the same environment tree in such a system. Since rerooting preserves the value of *assoc[v,a]*, executing *shallow[]* in any process cannot affect the values of the variables in the other processes so long as rerooting is indivisible.[3] Of course, if more than one process tries to do continuous shallow binding, then we will get a form of *thrashing* in which the system spends all of its time rerooting!

[3]We note that although a variable binding might be passed around in our scheme, there is only one copy of it, and hence this scheme is free from the "multiple copy update problem" of other fast access methods such as *caches*.

5. Dijkstra's Display as a Shallow Binding Technique

Dijkstra's *display* technique for fast variable access in lexically scoped languages [9] can be brought under the purview of our model, but we must first reformulate it. Each variable reference in an Algol 60 program is normally compiled as a reference to an offset within the current activation record for the block which defines its scope. One could conceptually[4] rewrite (i.e. do a source-to-source translation on) the Algol program in such a way that every block has exactly one variable a pointer variable which points to the current activation record for that block and all references to the original variables are mapped into offset references to these activation records via the pointer variables.[5] Call these introduced pointer variables *display variables* and notice that they are the only variables left in the program. Finally, give them names which correspond to their lexical depth in the program such as "`Display_variable_depth_1`" or "`Display_variable_depth_6`"; this naming assignment is consistent with the lexical scoping rule.[6]

We now see that these display variables are the ones being shallowly bound in Dijkstra's display vector; indeed, the display is simply a vector of memory locations each acting as a value cell for a display variable.

Rerooting is thus a value-preserving transformation on a tree environment structure and has nothing to do with how the structure was created; e.g. the interpreter could use either lexical or dynamic scoping, or even some combination of the two [1].

6. Roots

Our algorithm was discovered by pondering the Greenblatt Lisp machine proposal [3] which also describes a shallow scheme for handling general FUNARG's. His scheme does not reverse any pointers, but appends an "up/down" bit to each node in the environment tree which essentially indicates whether that link would have been reversed in our scheme. His less general method *must* continuously shallow bind because the pointers in the "down" path (the set of links which our model would have reversed) point in the wrong direction to be used by *assoc*. Our scheme both simplifies and generalizes his so that not only can shallow binding be implemented more uniformly, but we also get the serendipitous benefit of being able at any point to choose between shallow and deep binding.

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[4]Except for the fact that Algol 60 does not support pointer variables!

[5]We now assume that activation records live somewhere off in a heap.

[6]This naming convention tends to minimize the number of names required, e.g. if the value cells are implemented in special hardware registers as in the Burroughs B5500. Display variables can have any names consistent with the lexical scoping rule, including a distinct one for each block [4].

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